1 UNITS AND DIMENSIONAL FORMULA

1. PHYSICAL QUANTITIES

♦	PHYSICAL QUANTITIES
♦	UNITS AND DIMENSIONS

QUANTITY

Quantity means size, amount, magnitude or simply stated as the answer for 'how much?' or 'how many?'

PHYSICAL QUANTITY

Any quantity which can be 'measured' and expressed in terms of a number is called physical quantity.

Ex: Length, Area, Volume, Speed, Force, Energy etc.

Definition of Unit

A standard value used to measure a .physical quantity is called unit.

Ex: kg, m

Physical quantities are organized into various ways in various systems like FPS, CGS, MKS and SI. Now the scientists of all the countries follow SI for all their research work.

In SI, physical quantities are organized into three groups. They are:

- Fundamental physical quantities
- Supplementary physical quantities
- Derived physical quantities

SET OF FUNDAMENTAL QUANTITIES

It refers to a physical quantity that cannot be stated in any other physical quantity. One of its primary functions is to support other quantities.

Ex: [Mass, Length, Time]

In the above set, mass cannot be derived from the remaining physical quantities and similarly others, so it is a set of fundamental quantities.

Similarly, Electric current cannot be derived from the remaining physical quantities and similarly others, so it is a set of fundamental quantities.

In SI, there are seven fundamental physical quantities. They are Length, Mass, Time, Thermodynamic Temperature, Strength of Electric Current, Amount of substance and Luminous Intensity. Their units are called fundamental units or basic units.

SUPPLEMENTARY PHYSICAL QUANTITY

In SI, there are two supplementary physical quantities. They are

- Plane angle
- Solid angle

DERIVED PHYSICAL QUANTITY

Physical quantities derived from fundamental or supplementary physical quantities are called derived physical quantities and their units are called derived units.

FUNDAMENTAL UNITS

Units of fundamental physical quantities are called fundamental units. In SI, the fundamental units are given below:

Fundamental Quantity	S.I unit	Symbol
Length	metre	M
Mass	kilogram	kg
Time	second	s
strength of electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous Intensity	candela	Cd

Note: Fundamental units in some of the old systems are the following:

System	Fundamental Physical Quantities		
	Length	Mass	Time
FPS	Foot (ft)	Pound (lb)	Second (s)
CGS	Centimeter (cm)	Gram (g)	Second (s)
MKS	Metre (m)	Kilogram (kg)	Second (s)

SUPPLEMENTARY UNITS

Units of supplementary physical quantities are called supplementary units. In SI, units of supplementary physical quantities are the following:

Supplementary quantity	Unit	Symbol
Plane angle	Radian	rad
Solid angle	Steradian	Sr

1. PHYSICAL QUANTITIES

WORK SHEET LEVEL–I MAINS CORNER

SINGLE CORRECT ANSWER TYPE QUESTIONS

PHYSICAL QUANTITIES

1. Ampere is the unit of
   1) Resistance 2) Electric current 3) Conductance 4) All of these
2. Luminous Intensity is measured in terms of
   1) Candela 2) Mole 3) Kelvin 4) Radian
3. Mole is the unit of
   1) Distance 2) Time 3) Quantity of matter 4) Velocity
4. Dyne - second is the unit of
   1) Momentum 2) Force 3) Energy 4) Power
5. Light year is a unit of
   1)Time 2) Mass 3) Distance 4) Energy
6. The magnitude of any physical quantity
   1) Depends on the method of measurement
   2) Does not depend on the method of measurement
   3) Is more in SI system than in CGS system
   4) Directly proportional to the fundamental units of mass, length and time

LEVEL–II

PHYSICAL QUANTITIES

7. Newton–second is the unit of
   1) Velocity 2) Angular momentum
   3) Momentum 4) Energy
8. Electron volt is a unit of
   1) Charge 2) Potential difference
   3) Momentum 4) Energy
9. Unit of energy is
   1) J/s 2) Watt-day 3) Kilowatt 4) $g\text{-}cm/s^2$
10. Which of the following is smallest unit
    1) Millimetre 2) Angstrom 3) Fermi 4) Metre
11. Which one of the following pairs of quantities and their units is a proper match
    1) Electric field – Coulomb/m 2) Magnetic flux – Weber
    3) Power – Farad 4) Capacitance – Henry

LEVEL-III ADVANCED CORNER

SINGLE CORRECT ANSWER TYPE QUESTIONS

12. Which of the following quantity is expressed as force per unit area?
    1) Work 2) Pressure 3) Volume 4) Area

13. Which of the following is a derived unit?
1) Unit of mass 2) Unit of length 3) Unit of time 4) Unit of volume

14. dyne/cm^2 is not a unit of
1) Pressure 2) Stress 3) Strain 4) Young's modulus

15. Which of the following is not the unit of energy?
1) calorie 2) joule 3) electron volt 4) watt

LEVEL–IV

STATEMENT TYPE QUESTIONS

16. Statement-I: Mass, length and time are fundamental physical quantities.
Statement-II: They are independent of each other.
1) Both statements are true
2) Both statements are false
3) Statement 1 is true, Statement 2 is false.
4) Statement 1 is false, Statement 2 is true.

17. Statement-I: Density is a derived physical quantity.
Statement-II: Density cannot be derived from the fundamental physical quantities.
1) Both statements are true.
2) Both statements are false.
3) Statement 1 is true, Statement 2 is false.
4) Statement 1 is false, Statement 2 is true.

INTEGER TYPE QUESTIONS

18. 1000 kg/m^3=_____g/cm^3

MULTI CORRECT ANSWER TYPE QUESTIONS

19. Metre per second is the unit of
1) Speed 2) Velocity 3) Angular velocity 4) All of these

20. Which of the following belongs to derived quantities?
1) Force 2) Acceleration 3) mass 4) Current

21. Choose the correct statement
1) Number of fundamental quantities are limited
2) In M.K.S System there are 7 fundamental quantities
3) Number of fundamental quantities are unlimited
4) In S.I system there are two supplementary quantities

LEVEL–V

COMPREHENSION TYPE QUESTIONS

PASSAGE

Derived units are the units of derived physical quantities which are expressed in terms of fundamental units.

22. Kilowatt hour is the unit of
    1) Power 2) Energy 3) Pressure 4) Time
23. Pascal is the S.I unit of
    1) Impulse 2) Coefficient of viscosity
    3) Surface tension 4) Modulus of elasticity
24. $Kgms^{-1}$ is the unit of
    1) Force 2) Linear momentum
    3) angular momentum 4) Torque

MATRIX MATCH TYPE QUESTIONS

25.

Column-I	**Column-II**
a) Energy	p) coulomb
b) Frequency	q) watt
c) Charge	r) hertz
d) Power	s) erg
	t) joule

2. UNITS AND DIMENSIONAL FORMULA

	♦	DIMENSIONAL FORMULA
	♦	DIMENSIONAL LESS QUANTITIES

SYMBOLS FOR FUNDAMENTAL UNITS

The fundamental units of physical quantities in SI system are represented, conventionally, by capital letters as shown below:

1) Unit of length – L 2) Unit of mass- M
3) Unit of time - T
4) Unit of temperature- K or ºC
5) Unit of electric current – A or I
6) Unit of luminous intensity – Cd
7) Unit of quantity of matter – mol

The capital letters indicate the nature of fundamental units but not their magnitude.

The unit of a derived quantity may be obtained by properly compounding one or more of the above fundamental units.

DIMENSIONAL FORMULA

Dimensional formula and dimensions: In chemistry the molecular formula of a compound reveals its composition of the elements used for its preparation and their relative proportions. Similarly in physics the dimensional formula of a physical quantity indicates how and which fundamental units are involved in one unit of that quantity.

Dimensional formula: The expression showing the powers to which the fundamental units are to be raised to obtain one unit of a derived quantity is called dimensional formula of that derived quantity.

Dimensions: The dimensions of a physical quantity are the powers to which the fundamental units are to be raised to obtain one unit of that quantity.

Example 1: The product of length and breadth of a surface gives its area.

Area = length × breadth = $L \times L = L^2$ (or) $[M^\circ L^2 T^\circ]$.

Thus dimensional formula of area is L^2 (or) $[M^\circ L^2 T^\circ]$, dimensions of area are: 2 in length and zero each in mass and time. Unit of area is obtained by raising the unit of length to 2nd power.

Example 2: The product of length, breadth and height of a body gives its volume.

$\therefore$ Volume = length × breadth × height

= length × length × length = $L \times L \times L = L^3$ (or) $[M^\circ L^3 T^\circ]$

Thus dimensional formula of volume is L^3 (or) [$M^\circ L^3 T^\circ$].

Dimensions of volume are: 3 in length and zero each in mass and time. Unit of volume is obtained by raising the unit of length to third power.

Example 3: The quantity of mass and volume of a body gives its density

$\therefore$ Density = Mass /volume = Mass/ (length)^3 $= \frac{m}{l^3}$

Thus, dimensional formula of density is [$M^1 L^{-3} T^\circ$]

Dimensions of density are: 1 in mass, –3 in length and zero in time. Unit of density is obtained by raising the unit of mass to 1st power and unit of length to – 3 power.

In general dimensional formula of a quantity can be written as [$M^a L^b T^c$]. Thus, the quantity has a^{th} dimension in mass, b^{th} dimension in length, and c^{th} dimension in time.

Note: The dimension of fundamental quantities other than mass, length, time (i.e., Electric Current, Temperature, Amount of substance and Luminous intensity is 0.)

Method for finding dimensional formulae:

Step I: Write the formula of physical quantity.

Step II: Convert the formula in fundamental physical quantity.

Step III: Write the corresponding symbol for fundamental quantities.

Step IV: Make proper algebraic combination and get the result.

DIMENSIONAL FORMULAE OF SOME PHYSICAL QUANTITIES

S.No.	Physical Quantity	General formula	Dimensional formula	S.I. Unit
1	Area	Length × breadth	[$M^0L^2T^0$]	m^2
2	Volume	Length × breadth×height	[$M^0L^3T^0$]	m^3
3	Density	Mass / Volume	[$ML^{-3}T^0$]	kgm^{-3}
4	Speed, Velocity	Distance / time	[M^0LT^{-1}]	ms^{-1}
5	Acceleration	Change in Velocity/time	[M^0LT^{-2}]	ms^{-2}
6	Momentum	Mass × velocity	[MLT^{-1}]	$kgms^{-1}$
7	Force	Mass × acceleration	[MLT^{-2}]	newton(N)
8	Impluse	Force × time	[MLT^{-1}]	$kgms^{-1}$
9	Work, energy	Force × distance	[ML^2T^{-2}]	joule(J) or kg m^2s^{-2}
10	Power	Work /time	[ML^2T^{-3}]	watt(W)
11	Pressure	Force / area	[$ML^{-1}T^{-2}$]	Nm^{-2}(Pa)

12	Angular momentum	Momentum x arm	$[ML^2T^1]$	Kgm^2/s
13	Torque	Force x arm	$[ML^2T^2]$	Kgm^2/s^2

DIMENSIONAL CONSTANT

- The physical quantities which have dimensions and have a fixed value are called dimensional constants.

Ex: Gravitational constant (G), Planck's constant (h), Universal gas constant (R), Velocity of light in vacuum (c) etc.,

DIMENSIONAL LESS QUANTITIES

- Dimensionless quantities are those which do not have dimensions but have a fixed value.

(a) Dimensionless quantities without units.

Ex: Pure numbers, angle trigonometric functions, logarithmic functions etc.,

(b) Dimensionless quantities with units.

Ex: Angular displacement - radian, Joule's constant etc.,

DIMENSIONAL VARIABLES & DIMENSIONALESS VARIABLES

- Dimensional variables are those physical quantities which have dimensions and do not have fixed value.

Ex: velocity, acceleration, force, work, power etc.

Dimensionless variables:

- Dimensionless variables are those physical quantities which do not have dimensions and do not have fixed value.,

Ex: Specific gravity, refractive index, Coefficient of friction, Poisson's Ratio etc.,

Limitations of dimensional analysis method:

- Dimensionless quantities cannot be determined by this method. Constant of proportionality cannot be determined by this method. They can be found either by experiment (or) by theory.
- This method is not applicable to trigonometric, logarithmic and exponential functions.
- In the case of physical quantities which are dependent upon more than three physical quantities, this method will be difficult.
- In some cases, the constant of proportionality also possesses dimensions. In such cases we cannot use this system.

- If one side of equation contains addition or subtraction of physical quantities, we cannot use this method.

Quantities Having Same Dimensions.

S. No.	Dimension	Quantity
(1)	$[M^0L^0T^{-1}]$	Frequency, angular frequency, angular velocity, velocity gradient and decay constant
(2)	$[M^1L^2T^{-2}]$	Work, internal energy, potential energy, kinetic energy, torque, moment of force
(3)	$[M^1L^{-1}T^{-2}]$	Pressure, stress, Young's modulus, bulk modulus, modulus of rigidity, energy density
(4)	$[M^1L^1T^{-1}]$	Momentum, impulse
(5)	$[M^0L^1T^{-2}]$	Acceleration due to gravity, gravitational field intensity
(6)	$[M^1L^1T^{-2}]$	Thrust, force, weight, energy gradient
(7)	$[M^1L^2T^{-1}]$	Angular momentum and Planck's constant
(8)	$[M^1L^0T^{-2}]$	Surface tension, Surface energy (energy per unit area)
(9)	$[M^0L^0T^0]$	Strain, refractive index, relative density, angle, solid angle, distance gradient, relative permittivity (dielectric constant), relative permeability etc.
(10)	$[M^0L^2T^{-2}]$	Latent heat and gravitational potential
(11)	$[M^0L^2T^{-2}\theta^{-1}]$	Thermal capacity, gas constant, Boltzmann constant and entropy
(12)	$[M^0L^0T^1]$	$\sqrt{l/g}, \sqrt{m/k}, \sqrt{R/g}$, where l = length g = acceleration due to gravity, m = mass, k = spring constant
(13)	$[M^0L^0T^1]$	L/R, $\sqrt{LC}$, RC where L = inductance, R = resistance, C = capacitance
(14)	$[ML^2T^{-2}]$	$I^2Rt, \frac{V^2}{R}t, VIt, qV, LI^2, \frac{q^2}{C}, CV^2$ where I = current, t = time, q = charge, L = inductance, C = capacitance, R = resistance

2. UNITS AND DIMENSIONAL FORMULA

WORK SHEET LEVEL–I MAINS CORNER

SINGLE CORRECT ANSWER TYPE QUESTIONS

DIMENSIONAL FORMULA

1. To write the dimensions of a physical quantity a symbols used for mass, length and time respectively are:
   1) kg, m, s 2) kg, L, T 3) M, L, T 4) M, L, S
2. The physical quantity which have the same dimensional formula.
   1) frequency 2) angular velocity
   3) angular frequency 4) all of these
3. The dimensional equation for magnetic flux is
   1) $\left[ML^1T^2\right]$ 2) $\left[M^1L^1T^2\right]$ 3) $\left[MT^2I^{-1}\right]$ 4) $\left[ML^2T^{-2}I^{-1}\right]$
4. The physical quantity which has dimensional formula as that of $\frac{\text{Energy}}{\text{mass}\times\text{length}}$ is
   1) Force 2) Acceleration 3) Momentum 4) Energy
5. The dimensional formula of magnetic induction is
   1) $\left[LT^{-1}A^{-2}\right]$ 2) $\left[M^1L^1T^{-1}\right]$ 3) $\left[MT^{-2}A^{-1}\right]$ 4) $\left[M^1LT^{-1}A^1\right]$
6. The dimensional formula for Planck's constant (h) is
   1) $\left[M^1L^0T^{-1}\right]$ 2) $\left[M^1L^1T^{-2}\right]$ 3) $\left[M^0L^1T^{-3}\right]$ 4) $\left[M^1L^2T^{-1}\right]$
7. The dimensional formula of volume is
   1) $\left[M^0L^3T^0\right]$ 2) $\left[M^0L^3T^{-1}\right]$ 3) $\left[M^1L^1T^{-3}\right]$ 4) $\left[M^1L^1T^{-1}\right]$

DIMENSIONAL LESS QUANTITIES

8. A dimension less quantity
   1) Never has a unit 2) May have a unit
   3) Always has a unit 4) Never exist

LEVEL–II

DIMENSIONAL FORMULA

9. The dimensions of power are:
   1) $[ML^2/T^3]$ 2) $[ML^2/T^2]$ 3) $[ML^2/T]$ 4) $[ML/T^2]$
10. The physical quantity which have zero dimensions in mass is
    1) Momentum 2) Force 3) Area 4) Density

11. The dimensions of pressure are:
1) $[MLT^{-2}]$ 2) $[ML^{-1}T^{-2}]$ 3) $[ML^{-1}T^{-2}]$ 4) $[MLT^{2}]$
12. The dimensions of torque are:
1) $[ML^{2}T^{-2}]$ 2) $[MLT^{-2}]$ 3) $[ML^{-1}T^{-2}]$ 4) $[ML^{-2}T^{-2}]$
13. The expression $[ML^{-1}T^{-2}]$ represents
1) Momentum 2) Force 3) Pressure 4) None of these
14. The expression $[M^{1}L^{1}T^{-1}]$ represents
1) Length 2) Force 3) Power 4) Momentum
15. The expression $[M^{1}L^{1}T^{-1}]$ represents
1) Speed 2) Acceleration 3) Impulse 4) Work

DIMENSIONAL LESS QUANTITIES

16. The physical quantity that has no dimensions is:
1) angular velocity 2) linear momentum
3) angular momentum 4) strain
17. The dimensional formula of relative density is
1) $\left[ML^{-3}\right]$ 2) $\left[LT^{-1}\right]$ 3) $\left[MLT^{-2}\right]$ 4) Dimensionless

LEVEL-III ADVANCED CORNER

SINGLE CORRECT ANSWER TYPE QUESTIONS

18. Select the pair whose dimensions are same
1) Pressure and stress 2) Stress and strain
3) Pressure and force 4) Power and force
19. Dimensional formula $\left[ML^{-1}T^{-2}\right]$ does not represent the physical quantity
1)Young's modulus of elasticity 2) Stress
3)Strain 4) Pressure
20. Dimensional formula $\left[ML^{2}T^{-3}\right]$ represents
1)Force 2) Power 3) Energy 4) Work
21. Which pair has the same dimensions
1)Work and power 2) Density and relative density
3)Momentum and impulse 4) Stress and strain
22. Whose dimensions is $\left[ML^{2}T^{-1}\right]$
1) Torque 2) Angular momentum
3) Power 4) Work

23. The dimensions of universal gravitational constant are

1) $\left[M^{-2}L^{2}T^{-2}\right]$ 2) $\left[M^{-1}L^{3}T^{-2}\right]$ 3) $\left[ML^{-1}T^{-2}\right]$ 4) $\left[ML^{2}T^{-2}\right]$

LEVEL–IV

STATEMENT TYPE QUESTIONS

24. Statement I: Dimension of length in area is 2.

Statement II: Dimensions of physical quantity are the exponents to which the fundamental units are to be raised to get one unit of physical quantity.

1) Both Statements are true 2) Both Statements are false

3) Statement I is true. Statement II is false

4) Statement I is false. Statement II is true

25. Statement I: Plane angle is a dimensionless quantity.

Statement II: All unit less quantities are dimensionless.

1) Both Statements are true

2) Both Statements are false

3) Statement I is true, Statement II is false

4) Statement I is false, Statement II is true

INTEGER TYPE QUESTIONS

26. Force has _______ dimensions in length.

MULTI CORRECT ANSWER TYPE QUESTIONS

27. Which of the following pairs have same dimensions?

1) Torque and work 2) Momentum and impulse

3) Heat and work 4) Light year and wavelength

28. Which of the following quantities have the same dimension in time?

1) Acceleration 2) Force 3) Velocity 4) Work

29. Choose the correct statement

1) In force, mass have dimensions of one

2) In force, length have dimensions of one

3) In force, time have dimensions of two

4) In force, time have dimensions of one

LEVEL–V

COMPREHENSION TYPE QUESTIONS

PASSAGE

Power = $\frac{\text{work}}{\text{time}}$ and work = force $\times$ distance

30. Dimensions of length in power is

1) One 2) Two 3) Three 4) Four

31. Dimensions of time in power is

1)-1 2)-2 3)-3 4)-4

32. Dimensions of mass in power is

1) One 2) Two 3) Three 4) Four

MATRIX MATCH TYPE QUESTIONS

33. Matrix match

Column-I		Column-II	
a)	Dimensional formula of distance	p)	$\left[M^0L^0T\right]$
b)	Dimensional formula of angle	q)	$\left[ML^0T^0\right]$
c)	Dimensional formula of mass	r)	$\left[M^0LT^0\right]$
d)	Dimensional formula of time	s)	$\left[M^0L^0T^0\right]$

3. APPLICATION OF DIMENSIONAL ANALYSIS

♦	PRINCIPLE OF HOMOGENEITY
♦	APPLICATION OF DIMENSIONAL FORMULA

ANALOGY TO UNDERSTAND PRINCIPLE OF HOMOGENEITY

We know two apples + three apples = five apples and one banana + two bananas = three bananas but what about two bananas + three apples? Obviously we cannot add them as a single entity. Similarly, 5metre+3 metre=8metre, 3second+4second = 7 second but what about 3 metre+4second? Obviously we cannot add them as a single entity. Therefore we can add, subtract or equate only physical quantities of same kind or same nature. This principle is called the principle of homogeneity.

PRINCIPLE OF HOMOGENEITY:

According to this principle, a physical relation is dimensionally correct if the dimensions of fundamental quantities (mass, length and time) are the same in each and every term on either side of the equation. This principle is based on the fact that only quantities of the same kind (or nature) can be added or subtracted. Suppose three quantities A, B and C are related as A = B + C. We can add B and C if they are like quantities i.e., have the same nature. Similarly the sum of B and C can be equated to A if they are like quantities. i.e., In an equation like A = B + C, the quantities A, B and C must have the same nature.

APPLICATIONS OF DIMENSIONAL FORMULA

i) To check the correctness of a physical relation

The verification of given relation is based on the principle of homogeneity of dimensions i.e., each term on L.H.S as well as on R.H.S of the relation should possess same dimension.

Ex: The time period of simple pendulum is given as $T=2\pi\sqrt{l/g}$ where 'l' is length of pendulum and 'g' is the acceleration due to gravity. Verify the correctness of relation.

L.H.S: Dimensional formula of time period, $'T'=\left[M^0L^0T\right]$

R.H.S: 2π is a number. It has no dimensional formula

Dimensional formula of $l \rightarrow \left[M^0L^1T^0\right]$

Dimensional formula of $g \rightarrow \left[M^0L^1T^{-2}\right]$

Dimensional formula of $2\pi\sqrt{l/g} \rightarrow \sqrt{\frac{L}{LT^2}}=\sqrt{T^2}=\left[T^1\right]$

∴ The terms on LHS and RHS of the given relation have same dimensions. Hence the given relation is dimensionally correct.

(ii) Conversion of Units

Consider a physical quantity having the dimensional formula $M^aL^bT^c$, where a,b,c, are dimensions of mass, length and time respectively. In the 1st system the fundamental units are $M_1,L_1 \& T_1$ and numerical value of physical quantity is n_1. Hence the total magnitude of physical quantity can be expressed as $n_1\left[M_1^aL_1^bT_1^c\right]$. In the 2nd system, the fundamental units are M_2L_2 and T_2 and the corresponding numerical value of physical quantity is n_2. Hence the total magnitude of physical quantity can be expressed as $n_2\left[M_2^aL_2^bT_2^c\right]$.

Whatever may be the unit selected for measurement, the total magnitude of physical quantity is invariable. [i.e. not changed]

$\therefore n_1\left[M_1^aL_1^bT_1^c\right]=n_2\left[M_2^aL_2^bT_2^c\right]$

Thus if the numerical value in one system is known, the corresponding numerical value in other system can be calculated.

Ex: To find the number of ergs in one Joule.

In CGS and SI systems the units for energy are respectively erg and joule.

Dimensional formula of energy is $M\,L^2\,T^{-2}$

Comparing it with general dimensional formula $M^aL^bT^c$, one can write,

$\Rightarrow a=1,b=2,c=-2$

1st system: CGS system

$M_1=1g,\ L_1=1\text{ cm } \& T_1=1\text{ s},\ n_1=1\text{ erg}$

2nd system: SI system

$M_2=1kg=1000g, L_2=1m=100cm \& T_2=1s, n_2=1J$

$n_1 ergs=n_2 joule$

$\Rightarrow n_1\left[M_1^aL_1^bT_1^c\right]=n_2\left[M_2^aL_2^bT_2^c\right]$

$$\Rightarrow n_1=n_2\left[\frac{M_2}{M_1}\right]^a\left[\frac{L_2}{L_1}\right]^b\left[\frac{T_2}{T_1}\right]^c=1\left[\frac{1000}{1}\right]^1\left[\frac{100}{1}\right]^2\left[\frac{1}{1}\right]^{-2}=10^7$$

$\therefore 10^7$ ergs=1 joule.

(iii) Relationship between different physical quantities

This is also based on the principle of homogeneity of dimensions i.e. the physical quantities f same dimensions only can be equated, added or subtracted.

Ex: The centripetal force F acting on a particle moving uniformly in a circle may depend upon mass (m), velocity (v) and radius (r) of the circle. Derive the formula for F using the method of dimensions.

Let $F=Km^a v^b r^c ....[1]$

Where K is dimensionless constant of proportionality and a, b, c are the powers of m, v and r respectively to represent F.

Writing the dimensions various quantities in [1], we get

$$\left[M^1L^1T^{-2}\right]=M^a\left[LT^{-1}\right]^b L^c=\left[M^aL^bT^{-b}L^c\right]$$

$$\left[M^1L^1T^{-2}\right]=\left[M^aL^{b+c}T^{-b}\right]$$

Applying the principle of homogeneity of dimensions, we get a=1

$b+c=1,.......[2]$

$-b=-2$ or $b=2$

From [2], $c=1-b=1-2=-1$

Putting these values, we get $F=Km^1v^2r^{-1}$ or $F=K\dfrac{mv^2}{r}$

This is the required relation.

(iv) To find the dimensions of constants in a given relation:

Sometimes a physical relation contains constant. In order to find the dimensions of these constants, we can use the principle of homogeneity of dimensions.

Ex: The relation between pressure [P], volume [V] and temperature [T] of a gas is given by: $\left[P+\dfrac{a}{V^2}\right](V-b)=RT$

Here a and b are constants. It is desired to find the dimensions of a and b.

The quantity a/V^2 must represent a pressure since it is added to P.

$$\text{Dimension of 'P'}=\frac{[\text{Force}]}{[\text{area}]}=\frac{\left[MLT^{-2}\right]}{\left[L^2\right]}=\left[ML^{-1}T^{-2}\right]$$

$$\text{Dimension of 'V}^2\text{'}=\left[L^3\right]^2=\left[L^6\right]$$

$$\therefore \frac{[a]}{\left[L^6\right]}=\left[ML^{-1}T^{-2}\right] \text{ or } [a]=\left[ML^5T^{-2}\right]$$

Hence, the dimensions of a are $\left[ML^5T^{-2}\right]$

The constant b must represent a volume since it is subtracted from V.

$\therefore [b]=\left[L^3\right]$.

Hence the dimensions of b are $\left[L^3\right]$.

SOLVED EXAMPLES

1. A force F is given by $F=at+bt^2$, where t is time. What are the dimensions of a and b

Sol. From the principle of dimensional homogeneity $[F]=[at]\therefore$

$$[a]=\left[\frac{F}{t}\right]=\left[\frac{MLT^{-2}}{T}\right]=[MLT^{-3}]$$

Similarly, $[F]=[bt^2]\therefore[b]=\left[\frac{F}{t^2}\right]=\left[\frac{MLT^{-2}}{T^2}\right]=[MLT^{-4}]$

2. The dimensions of physical quantity X in the equation Force $=\frac{X}{\text{Density}}$ is given by

Sol. [X] = [Force] × [Density] = $[MLT^{-2}]\times[ML^{-3}]=[M^2L^{-2}T^{-2}]$.

3. E, m, l and G denote energy, mass, angular momentum and gravitational constant respectively, then the dimension of $\frac{El^2}{m^5G^2}$ are

Sol. [E] = energy $=[ML^2T^{-2}]$, [m] = mass = [M], [l] = Angular momentum = $[ML^2T^{-1}]$

[G] = Gravitational constant = $[M^{-1}L^3T^{-2}]$

Now substituting dimensions of above quantities in

$$\frac{El^2}{m^5G^2}=\frac{[ML^2T^{-2}]\times[ML^2T^{-1}]^2}{[M^5]\times[M^{-1}L^3T^{-2}]^2}=[M^0L^0T^0]$$

i.e., the quantity should be angle.

4. Conversion of Newton into Dyne.

Sol. The Newton is the S.I. unit of force and has dimensional formula $[MLT^{-2}]$.

So 1 N = 1 kg-m/sec^2

By using $n_2=n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c=1\left[\frac{kg}{gm}\right]^1\left[\frac{m}{cm}\right]^1\left[\frac{sec}{sec}\right]^{-2}$

$$=1\left[\frac{10^3gm}{gm}\right]^1\left[\frac{10^2cm}{cm}\right]^1\left[\frac{sec}{sec}\right]^{-2}=10^5$$

$\therefore$ 1 N = 10^5 Dyne

5. Conversion of gravitational constant (G) from C.G.S. to M.K.S. system the value of G in C.G.S. system is 6.67×10^{-8} C.G.S. units while its dimensional formula is $[M^{-1}L^3T^{-2}]$

So $G = 6.67 \times 10^{-8}\ cm^3/g\ s^2$

By using $n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c = 6.67\times10^{-8}\left[\frac{gm}{kg}\right]^{-1}\left[\frac{cm}{m}\right]^3\left[\frac{sec}{sec}\right]^{-2}$

$= 6.67\times10^{-8}\left[\frac{gm}{10^3 gm}\right]^{-1}\left[\frac{cm}{10^2 cm}\right]^3\left[\frac{sec}{sec}\right]^{-2} = 6.67\times10^{-11}$

$\therefore G = 6.67 \times 10^{-11}$ M.K.S. units

6. Conversion of 1 MW power on a new system having basic units of mass, length and time as 10kg, 1dm and 1 minute respectively is

Sol. $[P]=[ML^2T^{-3}]$

Using the relation $n_2 = n_1\left[\frac{M_1}{M_2}\right]^x\left[\frac{L_1}{L_2}\right]^y\left[\frac{T_1}{T_2}\right]^z$

$$=1\times10^6\left[\frac{1kg}{10kg}\right]^1\left[\frac{1m}{1dm}\right]^2\left[\frac{1s}{1min}\right]^{-3}$$

[As $1MW = 10^6 W$]

$=10^6\left[\frac{1kg}{10kg}\right]\left[\frac{10dm}{1dm}\right]^2\left[\frac{1sec}{60sec}\right]^{-3} = 2.16\times10^{12}$ unit

7. From the dimensional consideration, which of the following equation is correct

Sol. $T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R^3}{gR^2}} = 2\pi\sqrt{\frac{R}{g}}$ [As $GM = gR^2$]

Now by substituting the dimension of each quantity in both sides.

$[T] = \left[\frac{L}{LT^{-2}}\right]^{1/2} = [T]$

L.H.S. = R.H.S. i.e., the above formula is Correct.

8. If velocity v, acceleration A and force F are chosen as fundamental quantities, then the dimensional formula of angular momentum in terms of v, A and F would be

Sol. Given, v = velocity $=[LT^{-1}]$, A = Acceleration = $[LT^{-2}]$, F = force = $[MLT^{-2}]$

By substituting, the dimension of each quantity we can check the accuracy of the formula

[Angular momentum] = Fv^3A^{-2}

$[ML^2T^{-1}] = [MLT^{-2}][LT^{-1}]^3[LT^{-2}]^{-2} = [ML^2T^{-1}]$

L.H.S. = R.H.S. i.e., the above formula is Correct.

3. APPLICATION OF DIMENSIONAL ANALYSIS

WORK SHEET LEVEL–I MAINS CORNER

SINGLE CORRECT ANSWER TYPE QUESTIONS

PRINCIPLE OF HOMOGENEITY

1. In an equation like $A=B+C$, the quantities A, B and C.
   1) may have same nature 2) must have same nature
   3) must have different nature 4) all of these
2. In an equation like $A=B/C$, the quantities A, B and C.
   1) should have same nature 2) should have different nature
   3) may or may not have same nature 4) all of these
3. If $x = a + bt + ct^2$, where x is metres and t in seconds the unit of b is
   1) ms^{-2} 2) ms^{-1} 3) m^2s 4) m^2s^2
4. In the above problem (2) units of "c" is
   1) m 2) s^{-2} 3) ms^{-1} 4) ms^{-2}

APPLICATION OF DIMENSIONAL FORMULA

5. To find number of ergs in one joule ________ should be known.
   1) dimensional formula of energy
   2) dimensional formula of relative density
   3) dimensional formula of impulse
   4) dimensional formula of momentum
6. A physical quantity of same dimension can be
   1) added 2) equated 3) subtracted 4) all of these
7. To find the dimension of constant in a given relation we use
   1) principle of homogeneity
   2) dimensional formulae of physical quantities in the given relation
   3) both 1 and 2
   4) the value of the constant in the given relation
8. The unit of force is 1 kilo newton, the length is 1km and time is 100 second, what will be the unit of mass?
   1) 1 kg 2) 100 kg 3) 1000 kg 4) 10000 kg
9. If $1\ g\ cms^{-1}=x$ newton – second, then the number x is equal to
   1) 1×10^{-1} 2) 3.1×10^{-3} 3) 1×10^{-5} 4) 6×10^{-4}

LEVEL–II

PRINCIPLE OF HOMOGENEITY

10. In the relation $V=\frac{\pi}{8}\frac{pr^4}{nl}$, where the letters have their usual meanings. The dimensions of V are

1) $\left[M^0L^3T^0\right]$ 2) $\left[M^0L^3T^{-1}\right]$ 3) $\left[M^0L^{-3}T^{-1}\right]$ 4) $\left[M^1L^3T^0\right]$

11. If $V=\frac{A}{t}+Bt^2+ct^3$ where 'v' is velocity 't' is the time and A, B and C are constants then the dimensional formula of B is

1) $\left[M^0LT^0\right]$ 2) $\left[ML^0T^0\right]$ 3) $\left[M^0L^2T^1\right]$ 4) $\left[M^0LT^{-3}\right]$

12. If force F, Mass M and time T are chosen as fundamental quantities. Then the dimensional formula for length, is

1) $\left[FMT\right]$ 2) $\left[FM^{-1}T^2\right]$ 3) $\left[FL^2T^{-2}\right]$ 4) $\left[F^{-1}L^{-2}T^{-2}\right]$

13. If force F, Length L and time T are chosen as fundamental quantities, the dimensional formula for Mass is

1) $\left[FLT\right]$ 2) $\left[F^{-1}L^{-1}T^{-2}\right]$ 3) $\left[F^{-2}L^{-2}T^{-2}\right]$ 4) $\left[F^1L^{-1}T^2\right]$

APPLICATION OF DIMENSIONAL FORMULA

14. The distance travelled by a body in nth second is given by $s_n=u+\frac{a}{2}(2n\text{-}1)$ where u is initial velocity and a is acceleration. The dimensional formula of S_n are

1) [L] 2) $[LT^{-1}]$ 3) $[LT^{-2}]$ 4) $[L^{-1}T]$

15. The equation which is dimensionally correct among the following is

1) $V=u+at^2$ 2) $S=ut+at^3$ 3) $S=ut+\frac{1}{2}at^2$ 4) $t=S+av$

16. The largest mass (m) that can be moved by a flowing river depends on velocity (v), density (ρ) of river water and acceleration due to gravity (g). The correct relation is

1) $m\propto\frac{\rho^2v^4}{g^2}$ 2) $m\propto\frac{\rho v^6}{g^2}$ 3) $m\propto\frac{\rho v^4}{g^3}$ 4) $m\propto\frac{\rho v^6}{g^3}$

17. Conversion of 1 MW power on a new system having basic units of mass, length and time as 10kg, 1dm and 1 minute respectively is

1) 2.16×10^{12}unit 2) 1.26×10^{12}unit 3) 2.16×10^{10}unit 4) 2×10^{14}unit

LEVEL-III ADVANCED CORNER

SINGLE CORRECT ANSWER TYPE QUESTIONS

18. The Vander Waal's equation for a gas is $\left(P+\frac{a}{V^2}\right)(V-b)=nRT$ where P, V, R, T and n represent the pressure volume, universal gas constant, absolute temperature and number of moles of a gas respectively, 'a' and 'b' are constant. The ratio b/a will have the following dimensional formula:

1) $\left[M^{-1}L^{-2}T^{2}\right]$ 2) $\left[M^{-1}L^{-1}T^{-1}\right]$ 3) $\left[ML^{2}T^{2}\right]$ 4) $\left[MLT^{-2}\right]$

19. If pressure P, velocity V and time T are taken as fundamental physical quantities, the dimensional formula of the force is

1) $\left[PV^{2}T^{2}\right]$ 2) $\left[P^{-1}V^{2}T^{-2}\right]$ 3) $\left[PVT^{2}\right]$ 4) $\left[P^{-1}VT^{2}\right]$

20. From the dimensional consideration, which of the following equation is correct

1) $T=2\pi\sqrt{\frac{R^3}{GM}}$ 2) $T=2\pi\sqrt{\frac{GM}{R^3}}$ 3) $T=2\pi\sqrt{\frac{GM}{R^2}}$ 4) $T=2\pi\sqrt{\frac{R^2}{GM}}$

LEVEL–IV

STATEMENT TYPE QUESTIONS

21. Statement I: The dimensions of all the terms in an equation must be identical.
Statement II: This is called principle of homogeneity
1) Both Statements are true.
2) Both Statements are false.
3) Statement I is true. Statement is false.
4) Statement I is false. Statement II is true

INTEGER TYPE QUESTIONS

22. In the equation $x = ut + \frac{1}{2}at^2$, the dimension of length for RHS of the equation is_____

MULTI CORRECT ANSWER TYPE QUESTIONS

23. In the formula $v=a+bt$, where v is velocity and t is time,

1) Dimensional formula of a is $\left[LT^{-1}\right]$ 2) Dimensional formula of b is $\left[LT^{-2}\right]$

3) Dimensional formula of a is $\left[LT^{2}\right]$ 4) Dimensional formula of b is $\left[LT\right]$

24. The variation of pressure p with distance X is given by the relation $P = \frac{a+x}{b}$ then

1) Dimensional formula of a is [L]
2) Dimensional formula of a/b is $\left[M^1L^{-1}T^{-2}\right]$
3) Dimensional formula of b is $\left[M^{-1}L^2T^2\right]$
4) Dimensional formula of b/a is $\left[M^{-1}L^{-2}T^1\right]$

LEVEL–V

COMPREHENSION TYPE QUESTIONS

PASSAGE

If M_1, L_1, T_1 are the fundamental units in the first system and M_2, L_2, T_2 are the fundamental units in the second system, we can write

$N_1\left(M_1L_1T_1\right)=N_2\left(M_2L_2T_2\right)$

25. Which of the following are expressed correctly?

1) 1 newton = (1000g) (100 cm) $\left(s^{-2}\right)$ 2) 1 newton = (100g) (10 cm) $\left(s^{-2}\right)$
3) 1 newton = (10g) (1000 cm) $\left(s^{-2}\right)$ 4) 1 newton = (100g) (100 cm) $\left(s^{-2}\right)$

26. Express acceleration of $10\,cm\,s^{-2}$ in the unit $ft\,min^{-2}$.

(Hint: $1cm=3.281\times10^{-2}ft$)

1) $10\,cm\,s^{-2}=118\,ft\,min^{-2}$
2) $100\,cm\,s^{-2}=181\,ft\,min^{-2}$
3) $10\,cm\,s^{-2}=1181\,ft\,min^{-2}$
4) $10\,cm\,s^{-2}=1811\,ft\,min^{-2}$

27. If unit of force is 2N, that of length is 4 m and that of velocity is $6\,ms^{-1}$ the unit of mass is

1) $\frac{2}{9}$kg 2) $\frac{4}{3}$kg 3) $\frac{9}{2}$kg 4) $\frac{2}{5}$kg

MATRIX MATCH TYPE QUESTIONS

28.

Column-I	Column-II
a) Velocity	p) $[M^0LT^{-2}]$
b) Potential Energy	q) $[ML^{-1}T^{-2}]$
c) Kinetic Energy	r) $[ML^2T^{-2}]$
d) Torque	s) $[MLT^{-2}]$
	t) $[M^0LT^{-1}]$